Electricity intraday price modeling with marked Hawkes processes

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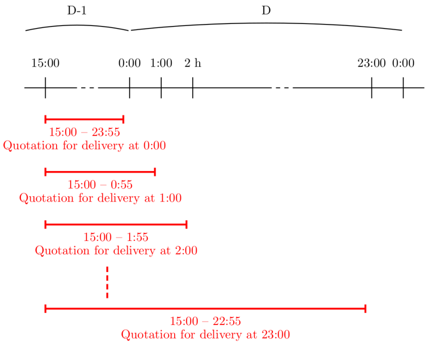
Motivation and objectives

- Renewable production is difficult to forecast when spot price is settled.
- Producers need to buy or sell electricity on the intraday market.
- Intraday markets allow to increase the value of some assets.
- Need for a price model that captures risks on the market to assess quality of strategies.
- Few literature on intraday markets modeling:
  - Favetto (2019); Graf von Luckner and Kiesel (2020) : order arrivals modeling
  - Kiesel and Paraschiv (2017) : econometric analysis
- We propose a price model with a focus on volatility modeling.

What are intraday markets?

EPEX Spot German intraday market, organized in continuous trading:

- Opens at 15:00 the day before;
- Possibility to buy/sell physical delivery contracts for the 24 periods 0:00–1:00, ..., 23:00–24:00;
- Closes 5 minutes before beginning of delivery.

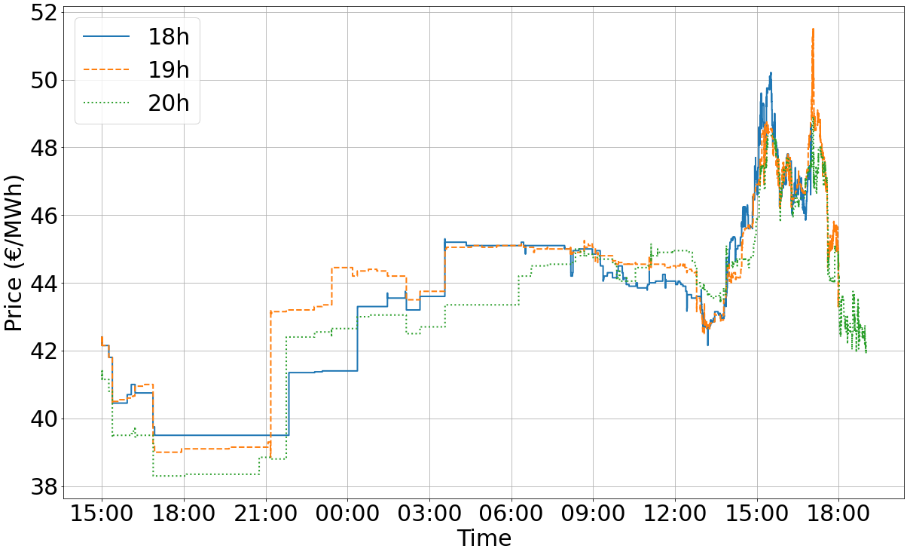


Data

- German electricity intraday mid-prices between July and September 2017 for products with a delivery period of one hour.
- Mid-prices built using order book data from EPEX Spot.
- Mid-prices sampled at the second frequency for simplicity (available at milliseconds frequency).
- Market opens at 3 p.m. the day before delivery and closes 5 minutes before delivery...
- Yet, one hour before delivery, cross-border trading is not possible anymore.
- Also, thirty minutes before delivery, transactions are only possible into each of the four control areas in Germany and not across them.

$\Longrightarrow$ We only consider prices until one hour before delivery.

Data: 2017-08-30



Outline

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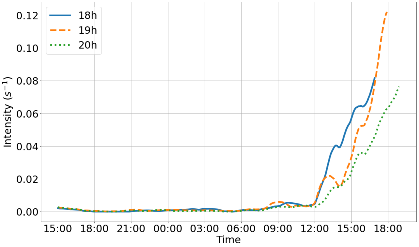
Increasing intensity of arrival price changing times



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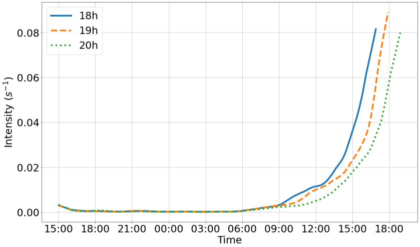

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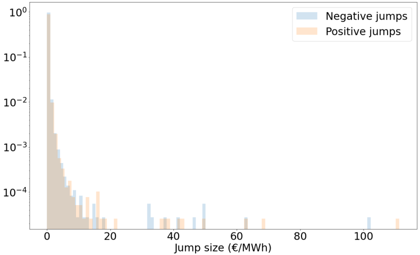
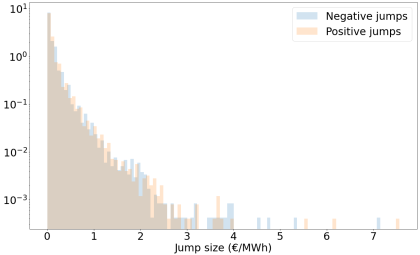
Estimated intensity of price changing times with an Epanechnikov kernel and a window of 300 seconds

- Quasi null activity at the beginning of the trading session...
- then an exponential increase near the end of the trading period.

Jump sizes distribution



All trading session



From 9 hours before maturity

Jump size distributions with a log scale on the y-axis (maturity 18h)

- Positive and negative jumps seem to have the same law (confirmed if we consider only the first two moments).
- Time dependency in the distribution of jumps with big jumps at the beginning, featuring a lack of liquidity.
- Also, stabilization of mean and standard deviation from 9 hours before maturity : From now on, one considers only data from 9 hours before maturity.

Volatility estimation

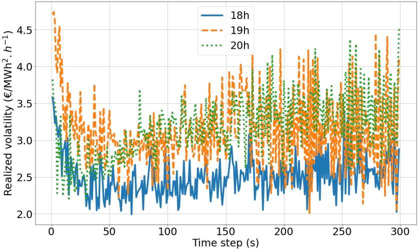
- Classical estimator of volatility of $f_t = \int_0^t \mu_s ds + \int_0^t \sigma_s dW_s$:

$$C(T, \Delta_n) = \frac{1}{T} \sum_{i=1}^{\lfloor \frac{T}{\Delta_n} \rfloor} \left(f_{i\Delta_n} - f_{(i-1)\Delta_n} \right)^2 \xrightarrow[\Delta_n \to 0]{} \frac{1}{T} \int_0^T \sigma_s^2 ds.$$

- One then wants to consider the highest frequency Δ_n^{-1}.
- Presence of microstructure noise in high-frequency financial data:
  - volatility estimator unstable when frequency is very high ;
  - mean reverting behavior of price.

Signature plot

$$\delta \mapsto C(T,\delta) = \frac{1}{T} \sum_{i=1}^{\lfloor \frac{T}{\delta} \rfloor} \left(f_{i\delta} - f_{(i-1)\delta} \right)^2$$



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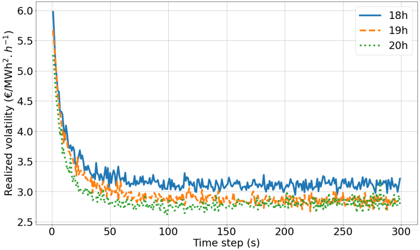

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- Same behavior than financial data, see Bacry et al. (2013).
- Instability at high-frequencies, fast decreasing then stabilization.

Outline

1. Empirical stylized facts
2. Model

Hawkes modeling (1/2)

- On $[0, T]$, price is modeled by $f_t = f_0 + \sum_{\tau_i^+ < t} J_i^+ - \sum_{\tau_i^- < t} J_i^-$ with
  - τ_i^+ (resp. τ_i^-) times of price increase (resp. decrease) with
  - $J_i^+ > 0$ (resp. $J_i^- > 0$) the associated jump sizes with same law J.
- Hawkes modeling for intensities of τ_i^+ and τ_i^-:

$$\lambda_t^+ = \mu\left(\frac{t}{T}\right) + \underbrace{\sum_{\tau_i^- < t} \varphi_{\exp}(t - \tau_i^-) J_i^-}_{\text{Cross excitation: impact of past downward jumps}}$$

$$\lambda_t^- = \underbrace{\mu\left(\frac{t}{T}\right)}_{\text{Baseline}} + \underbrace{\sum_{\tau_i^+ < t} \varphi_{\exp}(t - \tau_i^+) J_i^+}_{\text{Cross excitation: impact of past upward jumps}}$$

with
- $\mu : [0,1] \to \mathbb{R}_+ = t \mapsto \mu_0 e^{\kappa t}$: models the increasing intensity,
- $\varphi_{\exp} : \mathbb{R}_+ \to \mathbb{R} = t \mapsto \alpha e^{-\beta t}$, $\alpha, \beta > 0$, $\alpha \mathbb{E}(J) < \beta$: good candidate to represent the signature plot (Bacry et al. (2013)).

Hawkes modeling (2/2)

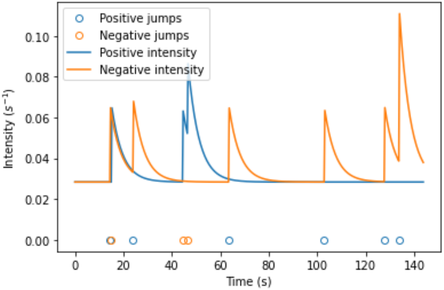


Figure: Intensity trajectory with constant baseline and jumps of size one

- Simple parameterisation with only four parameters.
- Tractable model with nice theoretical properties.
- A priori, allows to model the different characteristics of the prices.

Estimation of the parameters

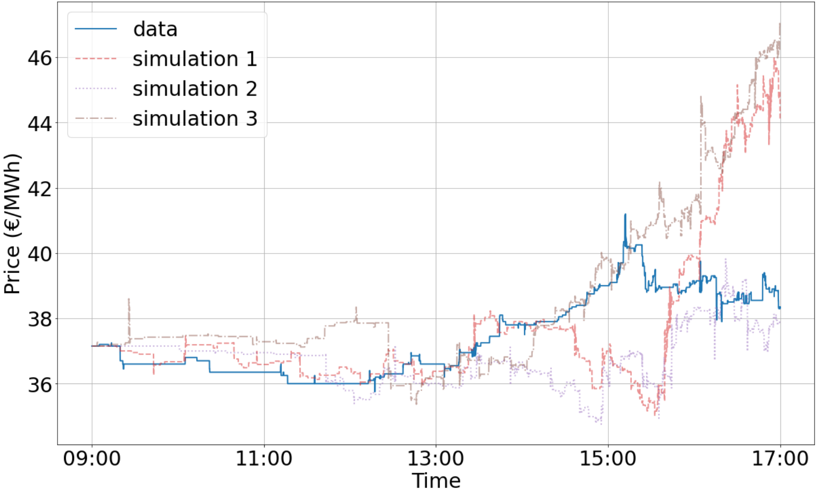
Estimation can be lead on the whole dataset by log-likelihood minimization.

Maturity	μ_0 (h^{-1})	κ	α (h^{-1})	β (h^{-1})	$\mathbb{E}(J)$	$\mathbb{E}(J^2)$	$\mathbb{E}(J)\frac{\alpha}{\beta}$
18h	2.49	3.51	864.39	237.30	0.13	0.066	0.47
19h	3.01	3.50	2344.97	639.64	0.13	0.061	0.48
20h	3.06	3.51	3100.46	859.11	0.13	0.058	0.47

- $\mathbb{E}(J)\frac{\alpha}{\beta}$ represents the percentage of endogenous price moves and seems to be the same for each hour.
- For μ_0, κ, $\mathbb{E}(J)$ and $\mathbb{E}(J^2)$, the estimated values are close to each other from one hour to another.

Simulation: Illustration for maturity 18h

Simulation with thinning algorithm Ogata (1981) bootstrapping jump sizes.



Signature plot

Explicit formulas for the signature plot, and for the expectation and the variance of prices as well.

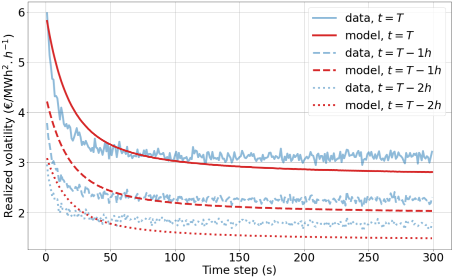


Table: Signature plots for the maturity 18h, also omitting the last hour and the last two hours

- Generalization of the results of Bacry et al. (2013): we include random jumps and time-dependent intensity baseline.
- Increasing of the signature plot when time approaches to delivery: Samuelson effect for each frequency.

Signature plot: asymptotics

Microscopic scale (left side of the SP): $\delta \to 0$

$$C^{micro}(t) = 2\mathbb{E}(J^2)\frac{\mathbb{E}\left(\int_0^t \lambda_s^+ ds\right)}{t}.$$

Macroscopic scale (right side of the SP): $\delta \to \infty$, $\frac{\delta}{t} \to 0$

$$C^{macro}(t) \sim \frac{2\mathbb{E}(J^2)}{\left(1+\frac{\alpha\mathbb{E}(J)}{\beta}\right)^2\left(1-\frac{\alpha\mathbb{E}(J)}{\beta}\right)}\frac{\int_0^t \mu(\frac{s}{T})ds}{t}.$$

When $t \to \infty$,

$$C(t,\delta) \sim \frac{2\mathbb{E}(J^2)\int_0^t \mu(\frac{s}{T})ds}{t\left(1-\frac{\alpha\mathbb{E}(J)}{\beta}\right)}\left(R^2+\left(1-R^2\right)\left(\frac{1-e^{-(\beta+\alpha\mathbb{E}(J))\delta}}{(\beta+\alpha\mathbb{E}(J))\,\delta}\right)\right)$$

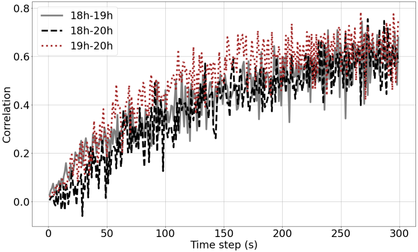
with $R^2 = \frac{1}{\left(1+\frac{\alpha\mathbb{E}(J)}{\beta}\right)^2}$.

Conclusion

- Highlighting of the presence of microstructure noise in intraday electricity markets;
- Proposition of a price model allowing to represent the different empirical stylized facts, in particular the signature plot;
- Closed formula for moments and signature plot (at different dates);
- Diffusive limit at macroscopic scale;
- Samuelson effect identified for each frequency and in the diffusive limit.

Perspectives

- A more complete analysis and modeling of jumps distribution (i.i.d. hypothesis strong).
- Are kernels exponential ?
- Multidimensional modeling for the different maturities.



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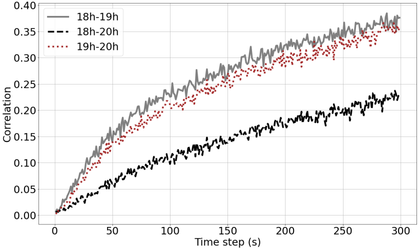

Average

Epps effect

Thank you for your attention.

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